short intro to frequency analysis (Fourier series)

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1 Rudiments of Frequency Analysis

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(Note: I am reusing some code from http://tinyurl.com/fourierpython)

In [1]:
```
%matplotlib inline
import matplotlib.pyplot as plt
from matplotlib.pyplot import plot, show
import numpy as np
from numpy import pi, sin, cos
import scipy

# Graphing helper function
def setup_graph(title='', x_label='', y_label='', fig_size=None):
    fig = plt.figure()
    if fig_size != None:
        fig.set_size_inches(fig_size[0], fig_size[1])
    ax = fig.add_subplot(111)
    ax.set_title(title)
    ax.set_xlabel(x_label)
    ax.set_ylabel(y_label)
```

1.1 The concepts of Amplitude, Frequency and Phase for a sinewave

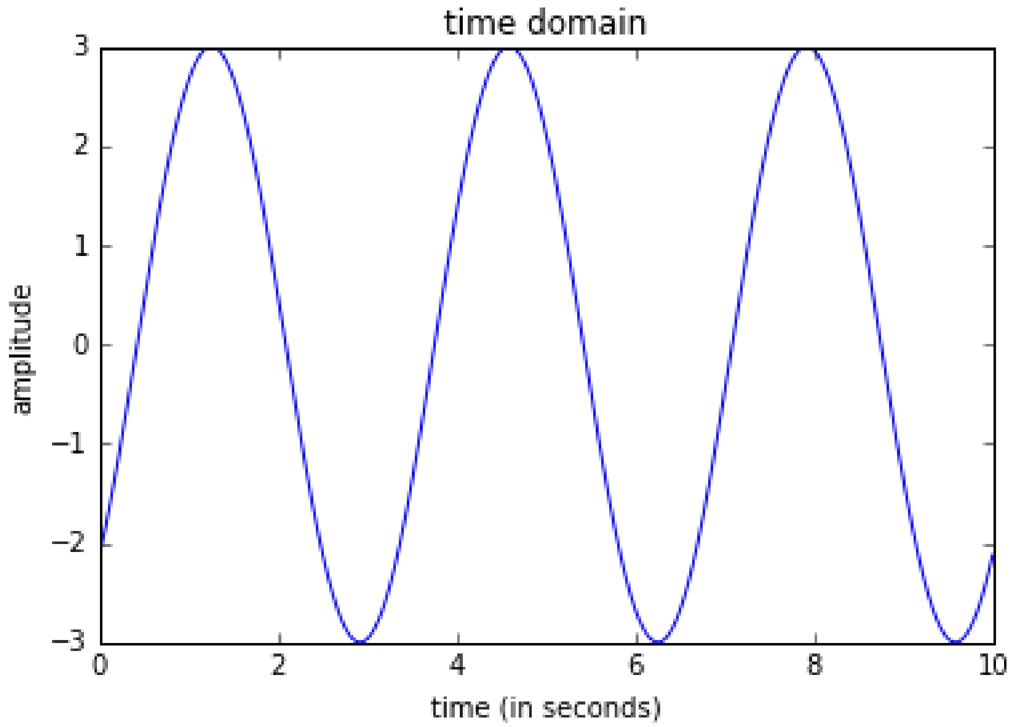
In [4]:
```
freq =  .3 #hz - cycles per second
phase = pi / 4.0
amplitude = 3


time_to_plot = 10 # second
sample_rate = 100 # samples per second
num_samples = sample_rate * time_to_plot

time = np.linspace(0, time_to_plot, num_samples)
signal = amplitude*sin((2*pi*freq*time) - phase)

setup_graph(x_label='time (in seconds)', y_label='amplitude', title='time domain')
plot(time, signal)
```

Out[4]: [<matplotlib.lines.Line2D at 0x7f1a0ae053c8>]

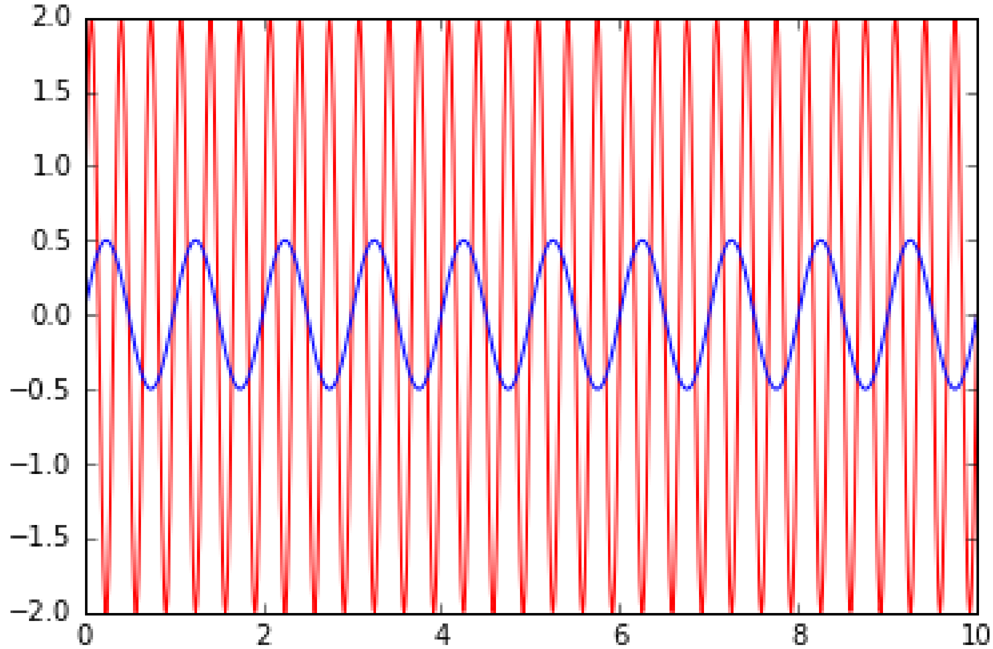



```
In [4]: freq1 = 3; amp1 = 2; phase1 = 0
        freq2 = 1; amp2 = 0.5; phase2 = 0

        y1 = amp1*sin((2*pi*freq1*time) - phase1)
        y2 = amp2*sin((2*pi*freq2*time) - phase2)

        plot(time, y1, 'r', time, y2, 'b')
```

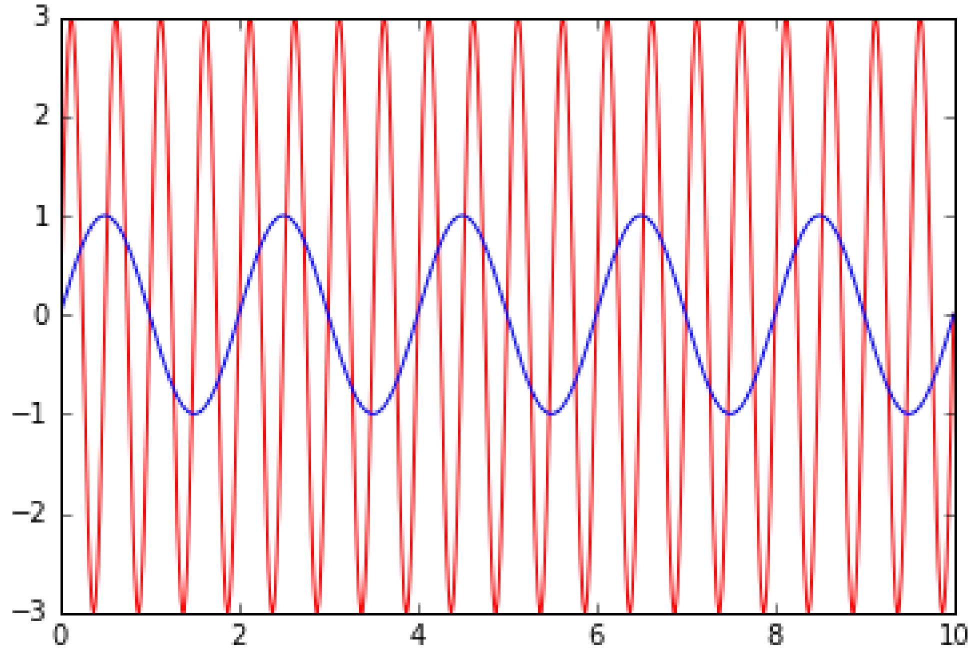
Out[4]: [<matplotlib.lines.Line2D at 0x7f3656c85358>,
 <matplotlib.lines.Line2D at 0x7f3656c85518>]

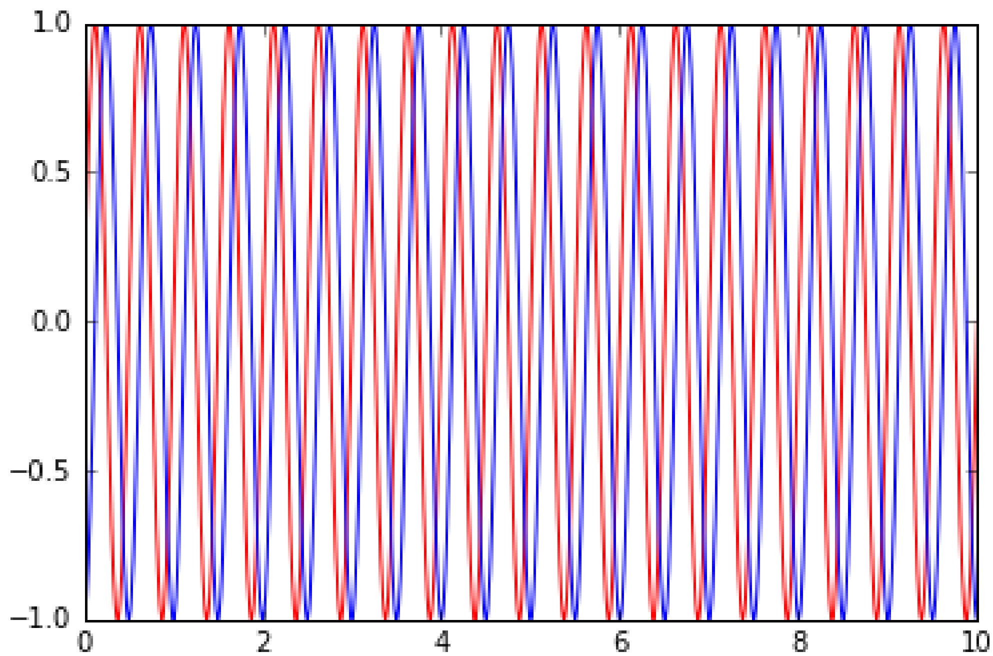


In [5]:
```
# Now using diffent sets of parameters:

params = ((3, 2, 0, 1, 0.5, 0),
          (1, 2, 0, 1, 2, pi/2))

for amp1, freq1, phase1, amp2, freq2, phase2 in params:
    y1 = amp1*sin((2*pi*freq1*time) - phase1)
    y2 = amp2*sin((2*pi*freq2*time) - phase2)
    plot(time, y1, 'r', time, y2, 'b')
    show()
```

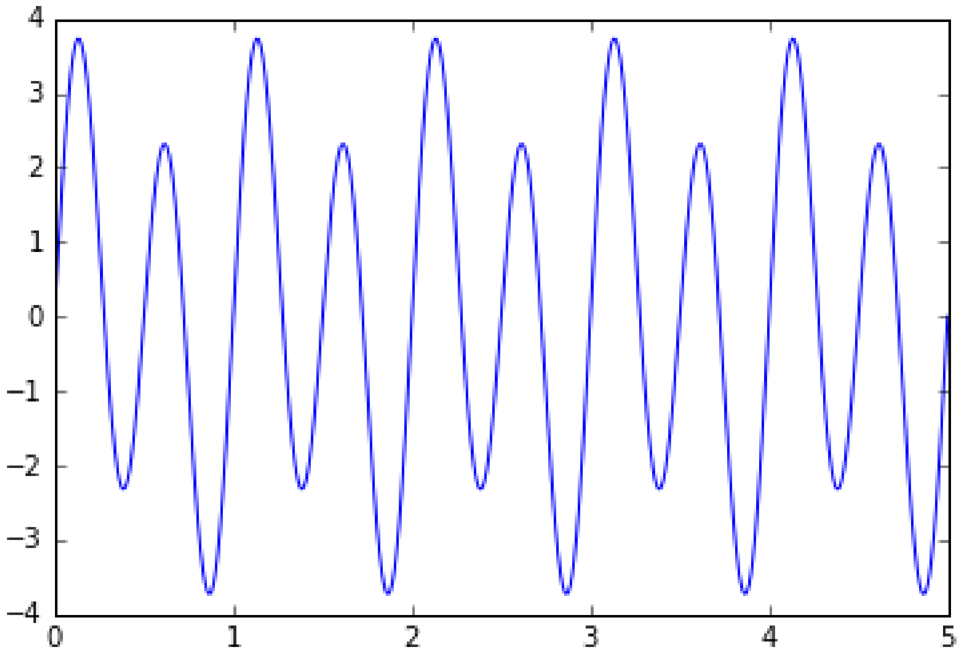

3
2
1
0
−1
−2
−3
0
2
4
6
8
10

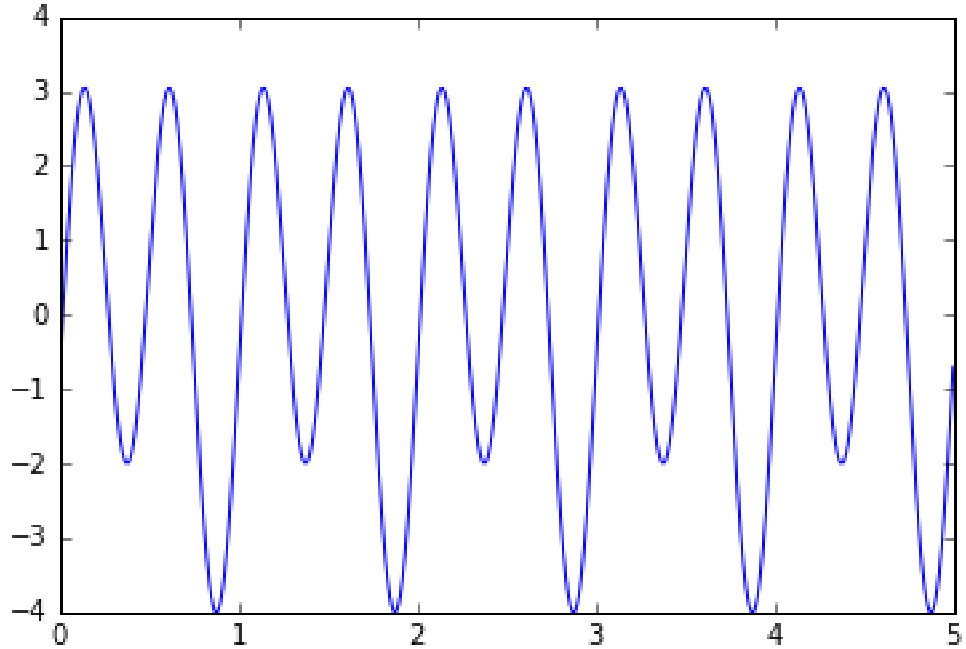

1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8
10

1.2 Summing sinewaves

```
In [15]: params = ((3, 2, 0, 1, 1, 0),
                   (3, 2, 0, 1, 1, pi/4))

         for a1, f1, p1, a2, f2, p2 in params:
             y1 = a1*sin((2*pi*f1*time) - p1)
             y2 = a2*sin((2*pi*f2*time) - p2)
             plot(time, y1 + y2)
             show()
```



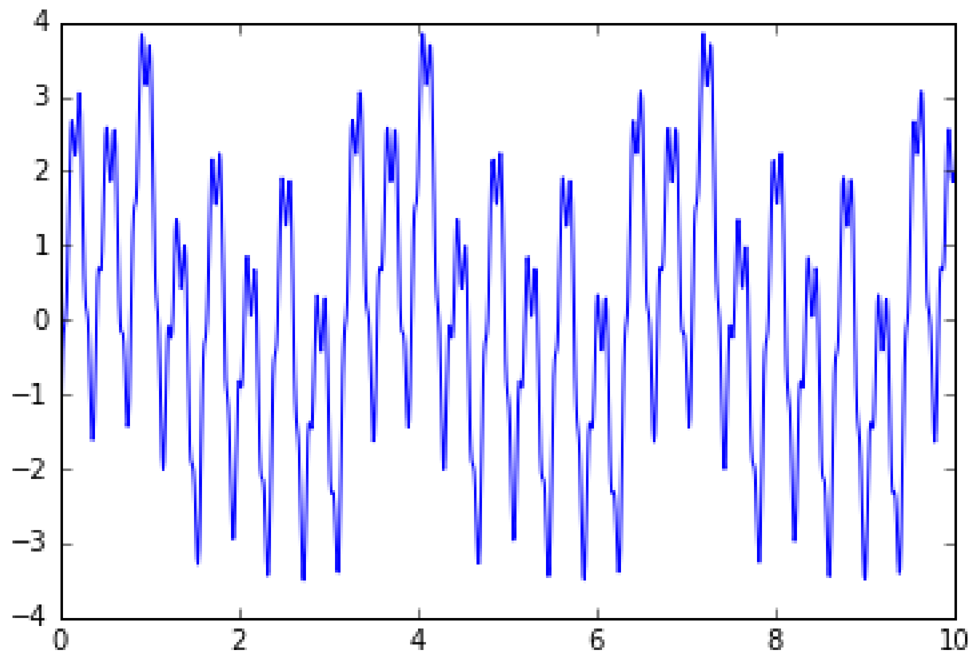



```
In [7]: # summing more than 2 sine waves (6 here)

        # Note; you can modify the following parameters and run the cell again:
        f = (1,2,4,8,16,32)
        a = (1,0.5,0.8,2,0.3,0.5)
        p = (0,1,0,1,0,0)

        y = np.zeros(len(time))
        for i in range(len(f)):
            y = y + a[i] * sin(2*f[i]*time-p[i])

        plot(time, y)
        show()
```



2 Decomposition

According to Fourier' theorem, any periodic signal can be expressed as the finite sum of sinewaves of varying amplitudes, phases and frequencies (frequencies which are multiples of the original frequency of the signal (1/period).

2.1 Creating basis functions

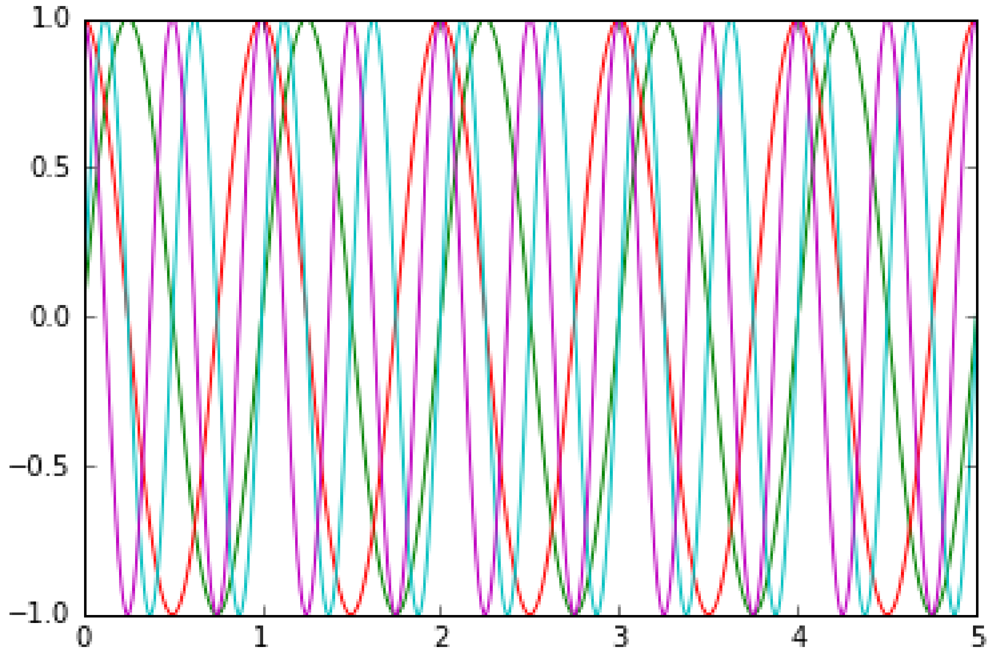
The Fourier basis comprises sines and cosines

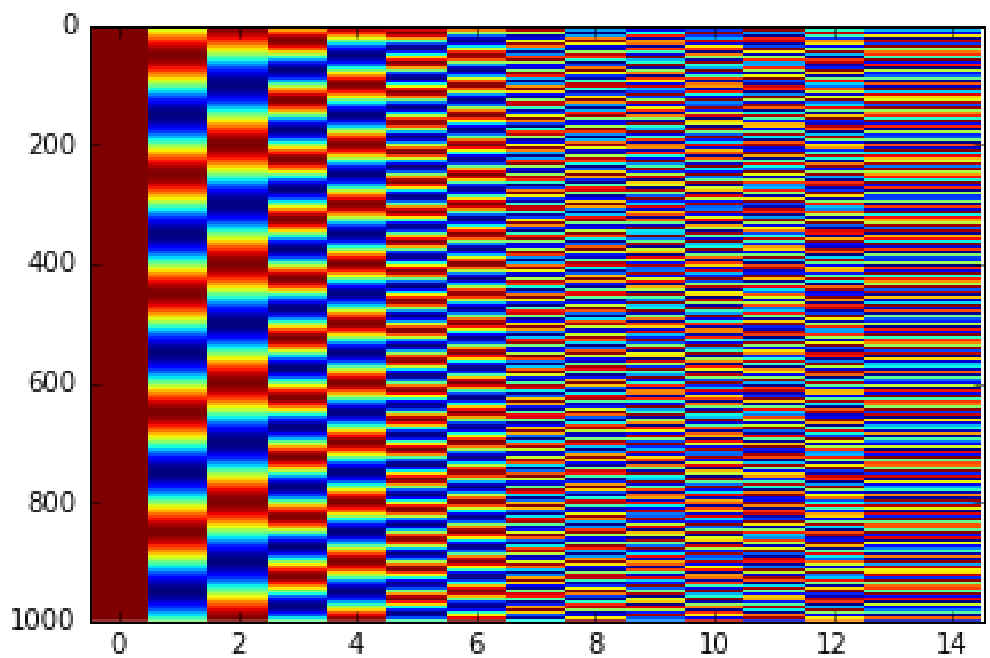
In [8]:
```
time = np.linspace(0, 5, 1000)
s0 = np.ones(len(time))
s1 = sin(2*pi*time); c1 = cos(2*pi*time)
s2 = sin(2*pi*2*time); c2 = cos(2*pi*2*time)
s3 = sin(2*pi*4*time); c3 = cos(2*pi*4*time)
s4 = sin(2*pi*8*time); c4 = cos(2*pi*8*time)
s5 = sin(2*pi*16*time); c5 = cos(2*pi*16*time)
s6 = sin(2*pi*32*time); c6 = cos(2*pi*32*time)
s7 = sin(2*pi*64*time); c7 = s7 = sin(2*pi*64*time)

X = np.column_stack((s0, s1, c1, s2, c2, s3, c3, s4, c4, s5, c5, s6, c6, s7, c7))

plt.plot(time, X[:,:5])
show()
```

```
plt.imshow(X, aspect='auto', interpolation='none')
show()
```

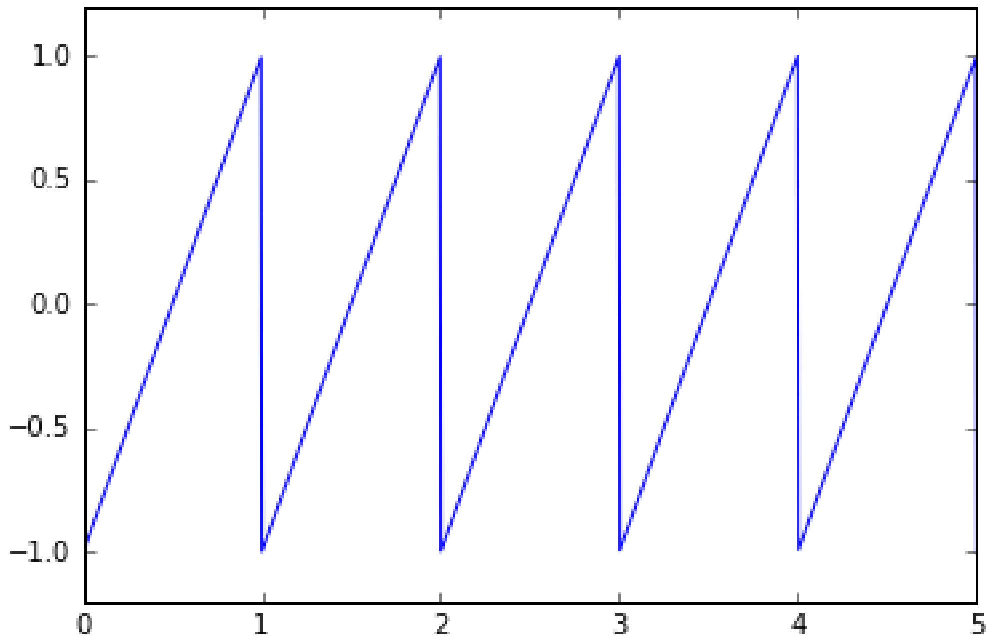




2.2 Decomposing a signal on the basis functions using multiple linear regression

Let us first generate a "sawtooth" signal

```
In [9]: from scipy import signal
        y = signal.sawtooth(2 * pi * time)
        plot(time, y)
        axes = plt.gca()
        axes.set_ylim([-1.2, 1.2])
        show()
```



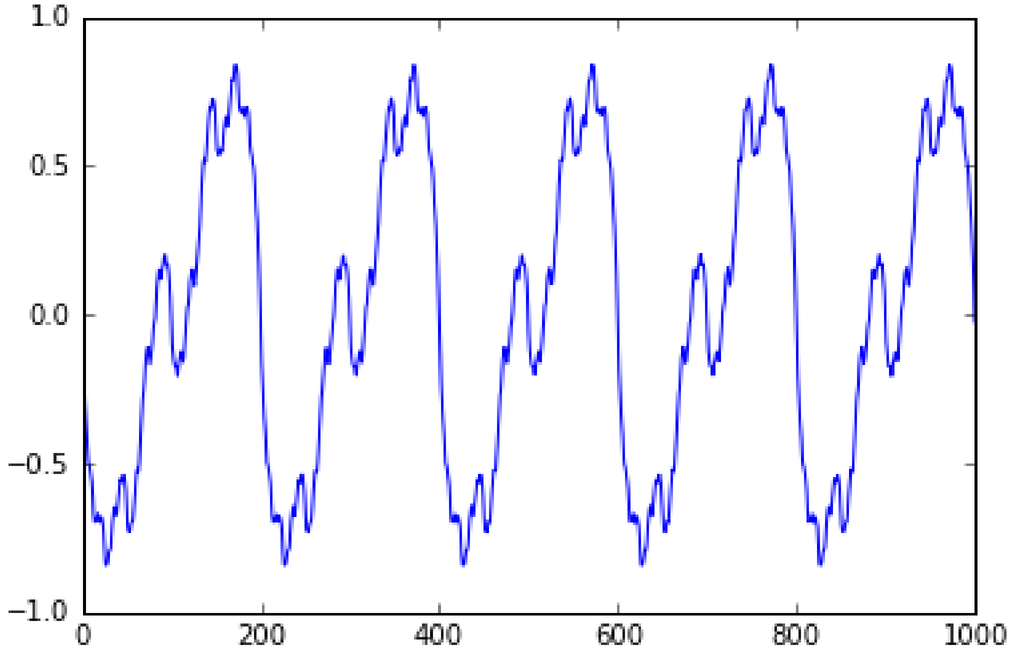
Let us now decompose this signal on the sinewave basis functions, using multiple linear regression (projection)

```
In [10]: coeff = np.linalg.lstsq(X,y)[0]   # least square estimates
         coeff
```

```
Out[10]: array([ -1.97628458e-03,  -6.35902481e-01,  -3.49075768e-03,
                 -3.18263795e-01,  -1.95312500e-03,  -1.58813477e-01,
                 -4.08008809e-03,  -7.95591338e-02,  -4.22151865e-03,
                 -3.97084691e-02,  -4.07893990e-03,  -1.98399520e-02,
                 -4.41487130e-03,   7.17544180e+12,  -7.17544180e+12])
```

The coefficients can be used to reconstruct a fitted signal that approximates the the original signal

```
In [11]: reconstructed_signal = X.dot(coeff)
         plot(reconstructed_signal)
         show()
```



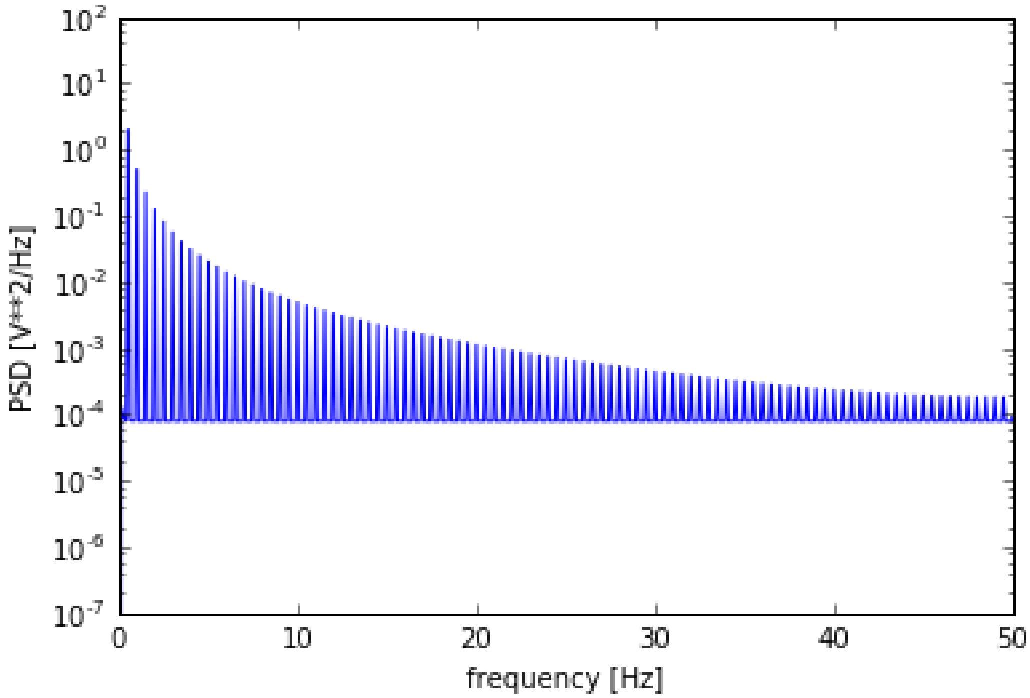

```
In [12]: #Computing and plotting the power spectrum
```

The discrete Fourier transform allows to get the coeffcients without explicitly creating the basis functions, making it easy to obtain a power spectrum, or periodogram of the signal.

```
In [13]: f, Pxx_den = signal.periodogram(y, sample_rate)

         plt.semilogy(f, Pxx_den)
         plt.ylim([1e-7, 1e2])
         plt.xlabel('frequency [Hz]')
         plt.ylabel('PSD [V**2/Hz]')

Out[13]: <matplotlib.text.Text at 0x7f36445c14a8>
```



A spectrogram shows a series a spectra computed over successive time-windows

```
In [14]: # example taken from matplotlib docs

         dt = 0.0005
         t = np.arange(0.0, 20.0, dt)
         s1 = np.sin(2*np.pi*100*t)
         s2 = 2*np.sin(2*np.pi*400*t)

         # create a transient "chirp"
         mask = np.where(np.logical_and(t > 10, t < 12), 1.0, 0.0)
         s2 = s2 * mask

         # add some noise into the mix
         nse = 0.01*rand(size(s2));

         x = s1 + s2 + nse;  # the signal
         NFFT = 1024;        # the length of the windowing segments
         Fs = int(1.0/dt);   # the sampling frequency

         # Pxx is the segments x freqs array of instantaneous power, freqs is
         # the frequency vector, bins are the centers of the time bins in which
         # the power is computed, and im is the matplotlib.image.AxesImage
         # instance
```

```
plt.plot(t, x)

Pxx, freqs, bins, im = plt.specgram(x, NFFT=NFFT, Fs=Fs, noverlap=900)
plt.show()
```